

Math 10460 - Honors Mathematics II

Homework 11b - Due Wednesday, April 13

You must show your work in all of the problems!!!

Recall that S^2 is the sphere, \mathbb{T} is the torus, and \mathbb{K} is the Klein bottle.

(5) Fill in the following chart

P	V	E	F	$\chi(P)$
Tetrahedron				
Cube				
Octahedron				
Dodecahedron				
Icosahedron				

(P is the polyhedron, V is the number of vertices, E is the number of edges, F is the number of faces, and $\chi(P)$ is the Euler characteristic of P .)

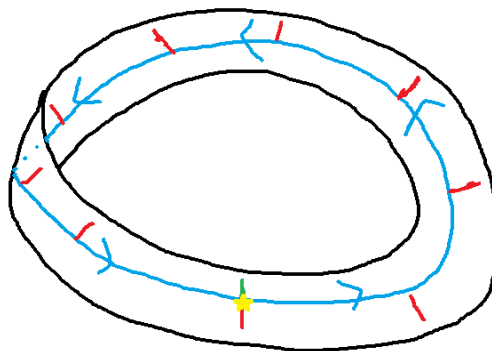
(6) Consider the standard pattern on a soccer ball:



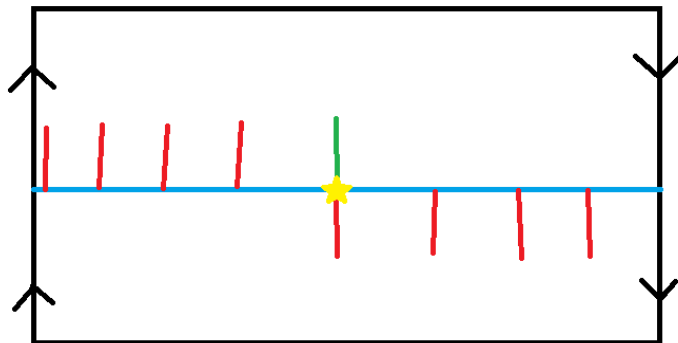
How many faces (pentagons and hexagons) does it have? How many edges? How many vertices? This can be thought of as a pattern of polygons on the sphere S^2 ; what is $\chi(S^2)$?

(7) Find the Euler characteristic of the Möbius band.

A closed loop along which orientation is reversed is called an *orientation-reversing loop*. Another definition of a non-orientable surface is a surface containing one of these loops. We can check if a loop is orientation reversing if when we traverse the loop starting on one side of it, when we get back to where we started, we're on the other side. Here's an example on the Möbius band:



and if we draw it on the plane model, it looks like this:



In both cases, we start on the side with the red lines at the star, then when we walk around the Möbius band and get back to the star, we are on the other side (at the green line).

- (8) Draw an orientation-reversing loop on the Klein bottle. (You can either draw it on a drawing of the surface, or on the plane model for the Klein bottle.)